Bayesian Parametric Approach for Multichannel Adaptive Signal Detection

Pu Wang, Hongbin Li, and Braham Himed

Abstract— This paper considers the problem of space-time adaptive processing (STAP) in non-homogeneous environments, where the disturbance covariance matrices of the training and test signals are assumed random and different with each other. A Bayesian detection statistic is proposed by incorporating the randomness of the disturbance covariance matrices, utilizing *a priori* knowledge, and exploring the inherent Block-Toeplitz structure of the spatial-temporal covariance matrix. Specically, the Block-Toeplitz structure of the covariance matrix allows us to model the training signals as a multichannel auto-regressive (AR) process and hence, develop the Bayesian parametric adaptive matched lter (B-PAMF) to mitigate the training requirement and alleviate the computational complexity. Simulation using both simulated multichannel AR data and the challenging KASSPER data validates the effectiveness of the B-PAMF in non-homogeneous environments.

Index Terms— Parametric adaptive matched lter, Bayesian detection, space-time adaptive signal processing, non-homogeneous environments.

I. INTRODUCTION

Traditional space-time adaptive processing (STAP) usually deals with homogeneous environments, where the test signal is assumed to share the same covariance matrix with the training signals [1]–[4].

To account non-homogeneous environments, a number of models have been proposed. One is the partially homogenous environment, which assumes the training signals share the covariance matrix with the test signal up to an unknown scaling factor [5], [6]. This model can be considered as a special case of the generalized eigenrelation (GER) [7]. Another one is the compound-Gaussian model, which assumes the training signals are a product of a texture (scaler) and a Gaussian vector. The texture is used to simulate power differences among the signals from range bins [8], [9]. More recently, a new class of non-homogeneous environments for adaptive signal detection emerges. This non-homogeneous environment is characterized by treating disturbance covariance matrices of both the test signal and training signals as random matrices and ensuring that they are different in probability one [10]–[14]. Following the Bayesian non-homogeneous model in [10], an adaptive matched lter has been derived by replacing the exact covariance matrix of the test signal by its maximum a

posteriori (MAP) estimate in the matched lter [10]. It has been shown that, by accounting the heterogeneity knowledge, the Bayesian adaptive matched lter (B-AMF) outperforms the standard AMF in the non-homogeneous environment. For applications of this Bayesian non-homogeneous model that employ space-time adaptive processing, the training requirement of the sample covariance matrix (SCM)-based B-AMF cannot be met due to the scarcity of the training signal. For example, with $J = 11$ spatial channels and $N = 32$ coherent pulses of the KASSPER dataset and assuming an instantaneous RF bandwidth of 500 KHz, $K = JN = 352$ training signals calls for the training range over a 200-km range, which is not practical [15], [16].

In this paper, while preserving the Bayesian non-homogeneous environment, we further explore the inherent Block-Toeplitz structure of the spatial-temporal covariance matrix which allows the block LDU decomposition [17], and hence enables the disturbances to be modeled as a multichannel auto-regressive (AR) process [17]–[23]. The resulting Bayesian parametric adaptive matched lter (B-PAMF) reduces the joint spatial-temporal whitening of the SCM-based B-AMF to successive spatial and temporal whitening. As a result, it facilitates the STAP in the non-homogeneous environments, and reduces the excessive training requirement of the B-AMF. Moreover, the B-PAMF is able to incorporate heterogeneities of the signals and utilize available *a prior* knowledge to the decision statistic. The effectiveness of the B-PAMF is veried by using the simulated multichannel AR data and the high delity KASSPER data [15].

II. SIGNAL MODEL

Assume J spatial channels, N temporal pulses, and K training range cells. The problem of interest is to detect a $JN \times 1$ multichannel signal $\mathbf{s}$ with *unknown* amplitude α in the presence of spatially and temporally correlated disturbance $\mathbf{d}_0$:

$$H_0: \quad \mathbf{x}_0(n) = \mathbf{d}_0(n), n = 0, 1, \cdots, N-1,$$
$$H_1: \quad \mathbf{x}_0(n) = \alpha \mathbf{s}(n) + \mathbf{d}_0(n), n = 0, 1, \cdots, N-1, \quad (1)$$

where $\mathbf{d}_0 = [\mathbf{d}_0^T(0), \mathbf{d}_0^T(1), \cdots, \mathbf{d}_0^T(N-1)]^T$, and $\mathbf{s}$ and $\mathbf{x}_0$ are similarly dened. In this paper, the signal model makes the following assumptions:

- **AS1 (Multichannel AR Process):** The disturbances in both test and training signals are modeled as a multichannel AR process [17]–[19]:

$$\mathbf{d}_k(n) = -\sum_{i=1}^{P} \mathbf{A}^H(i)\mathbf{d}_k(n-i) + \boldsymbol{\varepsilon}_k(n), k = 0, \cdots, K,$$
$$(2)$$

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1. REPORT DATE **MAY 2010**	2. REPORT TYPE	3. DATES COVERED **00-00-2010 to 00-00-2010**

4. TITLE AND SUBTITLE **Bayesian Parametric Approach for Multichannel Adaptive Signal Detection**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Air Force Research Laboratory,AFRL/RYRT,2241 Avionics Circle, Bldg 620,Dayton,OH,45433**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES
See also ADM002322. Presented at the 2010 IEEE International Radar Conference (9th) Held in Arlington, Virginia on 10-14 May 2010. Sponsored in part by the Navy.

14. ABSTRACT
This paper considers the problem of space-time adaptive processing (STAP) in non-homogeneous environments where the disturbance covariance matrices of the training and test signals are assumed random and different with each other. A Bayesian detection statistic is proposed by incorporating the randomness of the disturbance covariance matrices, utilizing a priori knowledge, and exploring the inherent Block-Toeplitz structure of the spatial-temporal covariance matrix. Speci cally the Block-Toeplitz structure of the covariance matrix allows us to model the training signals as a multichannel auto-regressive (AR) process and hence, develop the Bayesian parametric adaptive matched lter (B-PAMF) to mitigate the training requirement and alleviate the computational complexity. Simulation using both simulated multichannel AR data and the challenging KASSPER data validates the effectiveness of the B-PAMF in non-homogeneous environments.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**4**	

where $\mathbf{A}^H = \left[\mathbf{A}^H(1), \mathbf{A}^H(2), \cdots, \mathbf{A}^H(P)\right]$ denote the *unknown* multichannel AR coefficient matrix, and $\varepsilon_k(n)$ denote the $J \times 1$ temporally white but spatially colored noise vectors.

- **AS2 (Random Disturbance Covariance Matrix of Training Signals):** The noise vector $\varepsilon_k(n)$ is distributed as $\varepsilon_k(n) \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q})$, and the spatial covariance matrix $\mathbf{Q}$ follows an inverse complex Wishart distribution with degrees of freedom μ and mean $\bar{\mathbf{Q}}$:

$$p(\mathbf{Q}) = \frac{\left|(\mu - J)\bar{\mathbf{Q}}\right|^\mu}{\tilde{\Gamma}(J, \mu) \left|\mathbf{Q}\right|^{(\mu+J)}} e^{-(\mu-J)\operatorname{tr}\left(\mathbf{Q}^{-1}\bar{\mathbf{Q}}\right)}, \quad (3)$$

where $\tilde{\Gamma}(J, \mu) = \pi^{J(J-1)/2} \prod_{k=1}^{J} \Gamma\left(\mu - J + k\right)$ with Γ given by the Gamma function [24].

- **AS3 (Random Disturbance Covariance Matrix of Test Signal):** The noise vector in the test signal $\varepsilon_0(n) \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_0)$, and $\mathbf{Q}_0$, given $\mathbf{Q}$, has a complex Wishart distribution with degrees of freedom ν and mean $\mathbf{Q}$

$$p\left(\mathbf{Q}_0 | \mathbf{Q}\right) = \frac{\nu^{J\nu} |\mathbf{Q}_0|^{\nu-J}}{\tilde{\Gamma}(J, \nu)|\mathbf{Q}|^\nu} e^{-\nu\operatorname{tr}\left(\mathbf{Q}^{-1}\mathbf{Q}_0\right)}. \quad (4)$$

The multichannel AR process for the disturbances in both the test and training signals consists of two types of unknown parameters: one is the *deterministic* AR coefficient matrix $\mathbf{A}$, and the other is the *random* spatial covariance matrices $\mathbf{Q}$ and $\mathbf{Q}_0$. The available *a prior* knowledge is imposed on the mean of $\mathbf{Q}$, i.e., $\bar{\mathbf{Q}}$, which can be obtained from sources such as land-use maps, past measurements, etc [15]. It is said that the importance of the *a priori* knowledge $\bar{\mathbf{Q}}$ is controlled by parameter μ, while the the heterogeneities, i.e., the statistical differences between the test and training signals, are determined by parameter ν. Most importantly, $\mathbf{Q} \neq \mathbf{Q}_0$ with probability one, which ensures the environment non-homogeneous [10].

III. Bayesian Parametric Adaptive Matched Filter

By assuming perfect knowledge on parameters $\mathbf{A}$ and $\mathbf{Q}_0$, the solution to the problem of interest is the classical parametric matched filter (PMF) [17]:

$$T_{\text{PMF}} = \frac{\left|\sum_{n=P}^{N-1} \tilde{\mathbf{s}}^H(n)\mathbf{Q}_0^{-1}\tilde{\mathbf{x}}_0(n)\right|^2}{\sum_{n=P}^{N-1} \tilde{\mathbf{s}}^H(n)\mathbf{Q}_0^{-1}\tilde{\mathbf{s}}(n)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{\text{PMF}}, \quad (5)$$

where γ_{PMF} denotes the PMF threshold subject to a selected probability of false alarm, and the whitened steering vector and test signal are obtained by using the true AR coefficient matrix $\mathbf{A}$

$$\tilde{\mathbf{s}}(n) = \mathbf{s}(n) + \sum_{p=1}^{P} \mathbf{A}^H(p)\mathbf{s}(n - p), \quad (6)$$

$$\tilde{\mathbf{x}}_0(n) = \mathbf{x}_0(n) + \sum_{p=1}^{P} \mathbf{A}^H(p)\mathbf{x}_0(n - p). \quad (7)$$

The parametric AMF replaces the exact AR coefficient matrices $\mathbf{A}$ and the spatial covariance matrix $\mathbf{Q}_0$ in the PMF

statistic by their estimates (e.g., the maximum likelihood estimate (MLE) by using the training signals). For the non-homogeneous environment considered in this paper, due to the randomness of the spatial covariance matrix, we adapt a hybrid parametric AMF, which is denoted as the Bayesian PAMF (B-PAMF) by first obtaining the MLE of the deterministic AR coefficient matrix $\hat{\mathbf{A}}_{\text{ML}}$, then deriving a maximum *a posteriori* estimate (MAP) of the stochastic spatial covariance matrix $\hat{\mathbf{Q}}_{0,\text{MAP}}$ and finally replacing $\mathbf{A}$ and $\mathbf{Q}_0$ in the PMF statistic by their estimates.

A. MLE of $\mathbf{A}$

According to the signal model, the joint probability density function (pdf) of the training signals can be approximated (ignore the conditionality on the first P temporal vectors) as [18]

$$f\left(\mathbf{x}_1, \cdots, \mathbf{x}_K | \mathbf{A}, \mathbf{Q}\right) = \left[\frac{1}{\pi^J |\mathbf{Q}|} e^{-\operatorname{tr}\left(\mathbf{Q}^{-1}\mathbf{\Sigma}(\mathbf{A})\right)}\right]^{K(N-P)},$$

where

$$\mathbf{\Sigma}(\mathbf{A}) = \frac{1}{K(N-P)} \sum_{k=1}^{K} \sum_{n=P}^{N-1} \varepsilon_k(n)\varepsilon_k^H(n) \quad (8)$$

From **AS2**, we can remove the dependence of the above pdf on $\mathbf{Q}$ by integrating it over $\mathbf{Q}$:

$$
\begin{aligned}
f\left(\mathbf{x}_1, \cdots, \mathbf{x}_K | \mathbf{A}\right) &= \int f\left(\mathbf{x}_1, \cdots, \mathbf{x}_K | \mathbf{A}, \mathbf{Q}\right) p(\mathbf{Q}) d\mathbf{Q} \\
&= \frac{\left|(\mu - J)\bar{\mathbf{Q}}\right|^\mu}{\pi^{JK(N-P)}\tilde{\Gamma}(J, \mu)} \int |\mathbf{Q}|^{-L} e^{-\operatorname{tr}\left(\mathbf{Q}^{-1}\tilde{\mathbf{\Sigma}}\right)} d\mathbf{Q} \\
&= \frac{\left|(\mu - J)\bar{\mathbf{Q}}\right|^\mu \tilde{\Gamma}(J, L - J)}{\pi^{JK(N-P)}\tilde{\Gamma}(J, \mu)} |\tilde{\mathbf{\Sigma}}|^{-L+J}
\end{aligned}
\quad (9)
$$

where $L = \mu + J + K(N - P)$ and $\tilde{\mathbf{\Sigma}} = K(N - P)\mathbf{\Sigma}(\mathbf{A}) + (\mu - J)\bar{\mathbf{Q}}$. Therefore, finding the MLE of $\mathbf{A}$ is equivalent to minimizing the determinant of $\tilde{\mathbf{\Sigma}}$. Rewrite the matrix $\tilde{\mathbf{\Sigma}}$ as

$$
\begin{aligned}
\tilde{\mathbf{\Sigma}} &= K(N - P)\mathbf{\Sigma}(\mathbf{A}) + (\mu - J)\bar{\mathbf{Q}} \\
&= \hat{\mathbf{R}}_{xx} + \hat{\mathbf{R}}_{yx}^H \mathbf{A} + \mathbf{A}^H \hat{\mathbf{R}}_{yx} + \mathbf{A}^H \hat{\mathbf{R}}_{yy}\mathbf{A} + (\mu - J)\bar{\mathbf{Q}} \\
&= \left(\mathbf{A}^H + \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1}\right) \hat{\mathbf{R}}_{yy} \left(\mathbf{A}^H + \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1}\right)^H \\
&\quad + \hat{\mathbf{R}}_{xx} - \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1}\hat{\mathbf{R}}_{yx} + (\mu - J)\bar{\mathbf{Q}},
\end{aligned}
\quad (10)
$$

where

$$
\hat{\mathbf{R}}_{xx} = \sum_{k=1}^{K} \sum_{n=P}^{N-1} \mathbf{x}_k(n)\mathbf{x}_k^H(n), \ \hat{\mathbf{R}}_{yy} = \sum_{k=1}^{K} \sum_{n=P}^{N-1} \mathbf{y}_k(n)\mathbf{y}_k^H(n),
$$

$$
\hat{\mathbf{R}}_{yx} = \sum_{k=1}^{K} \sum_{n=P}^{N-1} \mathbf{y}_k(n)\mathbf{x}_k^H(n), \quad (11)
$$

with $\mathbf{y}_k(n) = \left[\mathbf{x}_k^T(n-1), \cdots, \mathbf{x}_k^T(n-P)\right]^T \in \mathbb{C}^{JP\times 1}$. Since $\hat{\mathbf{R}}_{yy}$ is nonnegative definite and the remaining terms $\hat{\mathbf{R}}_{xx} - \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1}\hat{\mathbf{R}}_{yx} + (\mu - J)\bar{\mathbf{Q}}$ do not depend on $\mathbf{A}$, it follows from (10) that

$$\tilde{\mathbf{\Sigma}} \geq \tilde{\mathbf{\Sigma}}|_{\mathbf{A}=\mathbf{A}_{\text{ML}}} = \hat{\mathbf{R}}_{xx} - \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1}\hat{\mathbf{R}}_{yx} + (\mu - J)\bar{\mathbf{Q}}, \quad (12)$$

where the MLE of $\mathbf{A}$ is given as

$$\hat{\mathbf{A}}_{\mathrm{ML}} = -\hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1}. \tag{13}$$

When $\tilde{\mathbf{\Sigma}}$ is minimized, the MLE $\hat{\mathbf{A}}_{\mathrm{ML}}$ will minimize any nondecreasing function including the determinant of $\tilde{\mathbf{\Sigma}}$.

B. MAP Estimate of $\mathbf{Q}_0$

The MAP estimate of $\mathbf{Q}_0$ requires the computation of the posterior distribution $f(\mathbf{Q}_0|\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_K)$:

$$f(\mathbf{Q}_0|\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_K) = \int f(\mathbf{Q}_0, \mathbf{Q}|\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_K)d\mathbf{Q}, \tag{14}$$

where

$$\begin{aligned} f(\mathbf{Q}_0, &\mathbf{Q}|\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_K) \\ &\propto f(\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_K|\mathbf{Q}_0, \mathbf{Q}) p(\mathbf{Q}_0|\mathbf{Q}) p(\mathbf{Q}) \\ &\propto |\mathbf{Q}_0|^{\nu-J} |\mathbf{Q}|^{-(L+\nu)} e^{-\operatorname{tr}(\mathbf{Q}^{-1}[\tilde{\mathbf{\Sigma}}+\nu\mathbf{Q}_0])}. \end{aligned} \tag{15}$$

As a result, (14) can be calculated as

$$\begin{aligned} \int f&(\mathbf{Q}_0, \mathbf{Q}|\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_K)d\mathbf{Q} \\ &\propto |\mathbf{Q}_0|^{\nu-J} \int |\mathbf{Q}|^{-(L+\nu)} e^{-\operatorname{tr}(\mathbf{Q}^{-1}[\tilde{\mathbf{\Sigma}}+\nu\mathbf{Q}_0])} d\mathbf{Q} \\ &\propto |\mathbf{Q}_0|^{\nu-J} |\tilde{\mathbf{\Sigma}} + \nu\mathbf{Q}_0|^{\mu+\nu+K(N-P)}. \end{aligned} \tag{16}$$

Taking the logarithm of the above equation, then taking the derivative with respect to $\mathbf{Q}_0$, and equaling to zero, we have

$$\begin{aligned} &\frac{\partial \ln f(\mathbf{Q}_0|\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_K)}{\partial \mathbf{Q}_0} \\ &\propto (\nu-J)\mathbf{Q}_0^{-1} - \nu(\mu+v+K(N-P))[\tilde{\mathbf{\Sigma}}+\nu\mathbf{Q}_0]^{-1} = 0, \end{aligned} \tag{17}$$

which suggests that, given $\mathbf{A}$, the estimate of $\mathbf{Q}_0$ is

$$\hat{\mathbf{Q}}_0 = \frac{(\nu-J)}{\nu(\mu+J+K(N-P))}\tilde{\mathbf{\Sigma}}. \tag{18}$$

Replacing $\mathbf{A}$ with $\hat{\mathbf{A}}_{\mathrm{ML}}$ of (13) in the above estimate (viz, $\tilde{\mathbf{\Sigma}}$), the MAP estimate of $\mathbf{Q}_0$ is

$$\begin{aligned} \hat{\mathbf{Q}}_{0,\mathrm{MAP}} = &\frac{(\nu-J)}{\nu(\mu+J+K(N-P))} \\ &\times \left[\hat{\mathbf{R}}_{xx} - \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1} \hat{\mathbf{R}}_{yx} + (\mu-J)\bar{\mathbf{Q}}\right]. \end{aligned} \tag{19}$$

It is seen that the MAP of $\mathbf{Q}$ is a linear combination of a standard estimate of $\mathbf{Q}$ as introduced in [18], [19] and the *a priori* knowledge $\bar{\mathbf{Q}}$. This linear combination has been seen before for non-parametric approaches [10], [25].

C. Bayesian PAMF

With the ML estimate of $\mathbf{A}$ and the MAP estimate of $\mathbf{Q}_0$, the adaptive version of the PMF in the heterogeneous environment can be derived as

$$T_{\mathrm{B\text{-}PAMF}} = \frac{\left|\sum_{n=P}^{N-1} \hat{\tilde{\mathbf{s}}}^H(n)\hat{\mathbf{Q}}_{0,\mathrm{MAP}}^{-1}\hat{\tilde{\mathbf{x}}}_0(n)\right|^2}{\sum_{n=P}^{N-1} \hat{\tilde{\mathbf{s}}}^H(n)\hat{\mathbf{Q}}_{0,\mathrm{MAP}}^{-1}\hat{\tilde{\mathbf{s}}}(n)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{\mathrm{B\text{-}PAMF}} \tag{20}$$

where $\gamma_{\mathrm{B\text{-}PAMF}}$ denotes the B-PAMF threshold subject to a selected probability of false alarm, $\hat{\mathbf{Q}}_{0,\mathrm{MAP}}$ is given by (19), and the whitened steering vector and the whitened test signal are obtained by using $\hat{\mathbf{A}}_{\mathrm{ML}}$ given by (13):

$$\hat{\tilde{\mathbf{s}}}(n) = \mathbf{s}(n) + \sum_{p=1}^{P} \hat{\mathbf{A}}_{\mathrm{ML}}^H(p)\mathbf{s}(n-p), \tag{21}$$

$$\hat{\tilde{\mathbf{x}}}_0(n) = \mathbf{x}_0(n) + \sum_{p=1}^{P} \hat{\mathbf{A}}_{\mathrm{ML}}^H(p)\mathbf{x}_0(n-p). \tag{22}$$

From (20), on one hand, it is seen that the B-PAMF performs successive whitening, i.e., temporal whitening followed by spatial whitening, as opposed to joint spatio-temporal whitening across all JN dimensions of the Bayesian AMF in [10]. On the other hand, compared with the standard PAMF [17], the B-PAMF incorporates the *a priori* knowledge, i.e., $\bar{\mathbf{Q}}$, and utilizes the heterogeneity parameter ν and the importance parameter of the *a priori* knowledge, i.e., μ into the estimate of the spatial covariance matrix $\mathbf{Q}$. Hence, it is allowed that the B-PAMF provides computational efficiency and mitigates training requirement, meanwhile exploiting the *a priori* knowledge and the heterogeneity to improve the performance of detection.

IV. NUMERICAL EXAMPLES

In this section, simulation results are provided to illustrate the performance of the B-PAMF. Specifically, we first test the B-PAMF by using simulated data which conforms to **AS1**, **AS2** and **AS3**, and then using the more challenging KASSPER dataset [15]. The disturbance signal is generated as a multichannel AR(2) process with AR coefficient $\mathbf{A}$ and a spatial covariance matrix $\mathbf{Q}$. The signal vector $\mathbf{s}$ corresponds to a uniform equispaced linear array with randomly selected normalized spatial and Doppler frequencies. The signal-to-interference plus noise ratio (SINR) is defined as

$$\mathrm{SINR} = |\alpha|^2 \mathbf{s}^H \bar{\mathbf{R}}^{-1}\mathbf{s}, \tag{23}$$

where $\bar{\mathbf{R}}$ corresponds to the assigned AR coefficient matrix $\mathbf{A}$ and the mean of the training spatial covariance matrix $\bar{\mathbf{Q}}$. For each Monte-Carlo trial, the spatial covariance matrix $\mathbf{Q}$ for the training signal is generated as an inverse Wishart distribution with mean $\bar{\mathbf{Q}}$, and then, given $\mathbf{Q}$, the spatial covariance matrix $\mathbf{Q}_0$ for the test signal is generated as a Wishart distribution with mean $\mathbf{Q}$.

We focus here on performance comparison between the B-PAMF and the standard PAMF [17] in the non-homogeneous environment. Figs. 1(a)-1(c) show the probability of detection versus the SINR for the B-PAMF and the standard PAMF in cases of different values of μ and ν, when $P = 2$, $J = 4$, $N = 16$, $K = 1$, and $P_f = 0.01$. It is seen that, in all three cases, the B-PAMF outperforms the standard PAMF. Specifically, for a fixed value of μ, a larger value of ν, more homogeneous environments, results in slightly improved performance of detection of the B-PAMF, from Fig. 1(a) to Fig. 1(b). On the other hand, for a fixed value of ν, increasing μ means more importance of the *a priori* knowledge of $\bar{\mathbf{Q}}$, which leads to wider performance gap between the B-PAMF and PAMF, as compared between Fig. 1(a) and Fig. 1(c).
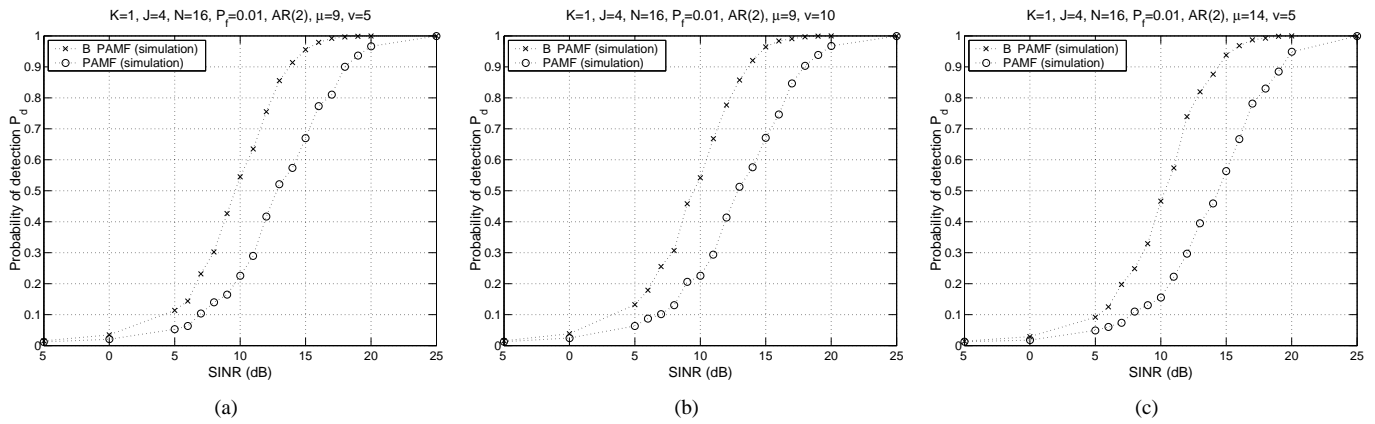
Fig. 1. Probability of detection versus SINR (a) when $P = 2$, $\mu = 9$, and $\nu = 5$; (b) when $P = 2$, $\mu = 9$, and $\nu = 10$; (c) when $P = 2$, $\mu = 14$, and $\nu = 5$.

V. CONCLUSION

A Bayesian parametric adaptive matched lter has been proposed by modeling the disturbances in the test and training signals as a multichannel AR process and simulating the heterogeneity between the training and test signals by introducing random disturbance covariance matrices. The B-PAMF admits successive temporal and spatial whitening, which reduces the computational complexity of the joint spatial-temporal whitening based adaptive detectors. The training requirement is also reduced. Simulation results validate the effectiveness of the B-PAMF.

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